ABSTRACTS

SIGA, 25-27 FEBRUARY 2014

MINI-COURSES

Alexander Schmitt (Freie Universität Berlin),

Title: Principal Bundles

Abstract, three talks

Part I The basic formalism of principal bundles.

Here, we work over a differentiable manifold. Topics: Definition of principal bundles - Associated fiber bundles - Extension and reduction of structure group - Examples

Part II Principal bundles over algebraic varieties.

The main technical problem in algebraic geometry is that the Zariski topology is, in general, not fine enough for principal bundles. Here, it is very instructive to look at the symplectic and the orthogonal group. Other topics: Descent theory - Adelic description of principal bundles - GAGA

Part III The Narasimhan-Seshadri theorem.

The Narasimhan-Seshadri theorem links representations of the fundamental group of a compact Riemann surface X to holomorphic or algebraic vector bundles on X. It is one of the main motivations to study principal bundles and has been vastly generalized since its publication.

We will highlight some aspects of the classical proof of Narasimhan and Seshadri.

Gian Pietro Pirola (Universita di Pavia),

Title: Theta characteristic on complex algebraic curves and applications

Abstract, three talks

1) Mumford and Kusner-Schmitt pairings. Introduced by Riemann, a theta characteristic on an algebraic curve C of genus g (or a spin structure on a Riemann surface) is a line bundle L that is a square root of the canonical bundle ω_C of $C : L^2 = \omega_C$. A theta characteristic often encodes important hidden geometric information. One main invariant is the parity of $h^0(L)$, the dimension of the space of the global sections of L and it is invariant by deformation. This was proved by Riemann using theta function and algebraically by Mumford. We present the latter approach [1] based on the residues as well as a generalization given in [2].

2) Theta characteristic and sub-canonical Weierstrass points. A point $p \in C$ on a smooth complex projective curve of genus g is sub-canonical if $\mathcal{O}_C(2g-2)p = \omega_C$ that is $L = \mathcal{O}_C(g-1)p$ is a theta characteristic. The locus $\mathcal{G}_g \subset \mathcal{M}_{g,1}$ described by pairs (C, p) as above has dimension $2g-1$ and consists of 3 irreducible components. Apart from the hyperelliptic component , $\mathcal{G}_g^{odd}$ and $\mathcal{G}_g^{even}$ depend on the parity of $h^0(C,(g-1)p)$, and their general points satisfy $h^0(C,(g-1)p) = 1$ and 2, respectively [3, 4]. We [5] study the subloci $\mathcal{G}_g^r$ of pairs (C,p) such that $h^0(C,(g-1)p) \geq r+1$. We provide a lower bound on their dimension, and we prove its sharpness for $r \leq 3$. As an application we give an existence result for triply periodic minimal surfaces in the 3−dimensional Euclidean space.

3) Theta characteristic and even Hurwitz scheme. In [6] spin structures are used to show that any compact Riemann surface covers $\mathbb{P}^1$ with map having only odd type of ramification points. In this case the monodromy is *even*, that is is contained in the alternating group. Coupling ideas of [1] and of [2], in [7] a partial compactification of these coverings has been found. This allows to study new geometrical aspects related with the moduli spaces of such coverings. We discuss some open problems about the irreducibility and the structure on the map from the *even* Hurwitz scheme to $\mathcal{M}_g$.

References

[1] D. Mumford, Theta-characteristic on an algebraic curve, *Ann. Sci. École Norm. Sup. (4)* **4** (1971), 181–192.
[2] R. Kusner and N. Schmidt, *The spinor representation of minimal surfaces*, 1995.
[3] M. Kontsevich and A. Zorich, Connected components of the moduli spaces of Abelian differentials with prescribed singularities, *Invent. Math.* **153**(3) (2003), 631–678.
[4] E. M. Bullock, Subcanonical points on algebraic curves, *Trans. Amer. Math. Soc.* **365**(1) (2013), 99–122.
[5] F. Bastianelli and G. P. Pirola, *Subcanonical points on projective curves and triply periodic minimal surfaces in the Euclidean space* (preprint).
[6] M. Artebani and G. P. Pirola, *Algebraic functions with even monodromy* Proc. Amer. Math. Soc. 133 (2005), 331-341.
[7] J. Naranjo and G. P. Pirola, *A globalisation of the Mumford and Kusner-Schmitt pairings and odd ramification coverings* (preprint).

TALKS

TUESDAY 25th

Leticia Brambila-Paz (CIMAT)

Title: On Chow stability.

Abstract: In the last decades there have been introduced different concepts of stability for projective varieties. In a joint work with Hugo Torres we give a natural and intrinsic criterion for the Chow, and Hilbert, stability for irreducible curves in a projective space $\mathbb{P}^n$. In this talk I will show how we use such criterion to describe the quotient stack of such curves with genus $g \geq 3$ and degree $d > g + n - \left\lfloor \frac{g}{n+1} \right\rfloor$.

Martha Bernal (U. de Chiapas)

Title: The Cox Ring of $\bar{M}_{06}$.

Abstract: The Cox ring of $\overline{\mathcal{M}_{0,6}}$ is generated by the sections of the boundary divisors and the Keel-Vermeire divisors. We compute a presentation for $\mathrm{Cox}(\overline{\mathcal{M}_{0,6}})$ using these generators and explore how to get the Mori chamber decomposition of the effective cone $\overline{\mathrm{Eff}}(\overline{\mathcal{M}_{0,6}})$.

Abel Castorena (CCM, Michoacan)

Title: Some applications of limit linear series.

Abstract: We give the definition of limit linear series for higher rank and we discuss some problems of Vector bundles over algebraic curves. We explain how the Limit linear series can help to solve some problems related with Brill-Noether theory.

WEDNESDAY 26th

Osbaldo Mata Gutierrez (CIMAT)

Title: On substacks of moduli stack of vector bundles over a curve.

Abstract: Let $B(n,d)$ the moduli stack of vector bundles of rank n and degree d over an algebraic curve X. To study $B(n,d)$, we define, using the (k,l)-stability, some substacks $B^{(k,l)}(n,d)$ of $B(n,d)$. In this talk we will describe $B^{(k,l)}(n,d)$ and explain some of its geometric properties. Furthermore, we will show how this properties could give more information of the moduli spaces $M(n,d)$ and $M^s(n,d)$ of stable and semistable vector bundles.

Hugo Torres (CIMAT)

Title: Stability vs linear stability.

Abstract: Let C be an irreducible smooth projective curve of genus $g \geq 2$ and (L,V) a generated linear series. The Dual Span Bundle $M_{V,L}$ associated to this data is the kernel of the evaluation morphism

$$ev : 0 \to M_{V,L} \to V \otimes \mathcal{O}_C \stackrel{ev}{\to} L \to 0.$$

In the recently decades, the stability of $M_{L,V}$ have been studied from different point of view. In this talk we give an equivalence between the stability of dual spam bundle and the linear stability of the linear series (L,V).

Armando Sanchez (U. de Oaxaca)

Title: Finite group actions on Abelian varieties with non-Abelian smooth quotients.

Abstract: The objective of this work is the construction and study of non-Abelian smooth quotients of an Abelian variety under the action of a finite group.

Let A be an Abelian variety over an algebraically closed field k with a structure of Z[G]-module with G a finite reductive group. For any $[\sigma] \in H^1(G,A)$, we define a natural action of G on the underlaying variety of A and, if this action on A is free of fixed points, we prove that the quotient variety Q has trivial canonical bundle, i.e. Q is Calabi-Yau, if and only if the induced action of G in $\wedge^{top} H^1(A, \mathcal{O}_A)$ is trivial. Further, if k=C we prove that fundamental group of Q is isomorphic to the extension defined by $\Delta([\sigma]) \in H^2(G, H^1(A,\mathbb{Z})))$ where $\Delta : H^1(G, JY) \longrightarrow H^2(G, H^1(A,\mathbb{Z})))$ is a connecting morphism in cohomology.

We give examples of this situation in which A is the Picard variety of a smooth variety V with a proper discontinuous action of G, i.e. the quotient morphism $\pi : V \longrightarrow V/G$ is etale. When V is a smooth curve, we prove that the varieties Q are naturally subschemes of certain moduli of semistable vector bundles on V/G, and also, that the moduli space of flat vector bundles on Q has a natural structure of subscheme of the moduli space of flat vector bundles on V/G.

Laura Hidalgo (UAM-I) **Title:** Compactification of the Prym fibre $P : R_4 \to A_3$.

Abstract: The main of this talk is to give a description of the fibre of the Prym map $\mathfrak{P} : \overline{\mathfrak{R}}_4 \to \mathfrak{A}_3$ on an principally polarized abelian variety (p.p.a.v.) of the form (A, Θ), where $A = J_1 \times J_2$ and $\Theta = J_1 \times \Theta_2 + \Theta_1 \times J_2$ where J_k is a Jacobian variety of dimension k.

The talk is divided in three parts, in the first we present the introduction to the problem; in the second, we give the description of allowable double cover in the Beauville sense; and in the last part, we give the description of $\mathfrak{P}^{-1}(A)$.

THURSDAY 27th

Jesus Romero (U. de Guerrero)

Title: Abeliantes and jacobianas.

Abstract: The abeliant is a matrix constructed by a polynomial rule. We will see how abeliants can be used to give a construction of the Jacobian of a nonsingular projective algebraic curve over an algebraically closed field by means of the abstract Abel map.

Lilia Alanís (CIMAT),

Title: Polar Filtration on the Milnor Fiber of an isolated singularity and the cup product on its cohomology.

Abstract: In 1987, J.H.M. Steenbrink and Joseph Zucker studied the relation between the Milnor fiber of an isolated singularity curve and the polar filtration induced by the Puiseux exponents of its polar curve. In 1990 Wilfred Homann generalized the same results for holomorphic functions in $\mathbb{C}^{n+1}$ with an isolated singularity. Using these results we want to study the cup product on the cohomology of its general fiber and give a description via the polar filtration.

Claudia Reynoso (DEMAT, U. de Guanajuato)

Title: Singular schemes of holomorphic foliations on $\mathbb{CP}^2$

Abstract: In this talk we will consider the space of foliations on CP2 and we will construct locally closed spaces with foliations with a particular singular scheme.